Carbon pricing and industrial competitiveness: Border adjustment or free allocation?

Robert A. Ritz*
Energy Policy Research Group
Judge Business School
University of Cambridge

This version: May 2022

Abstract

To mitigate concerns about carbon leakage and industrial competitiveness, cap-and-trade systems have typically relied on the free allocation of carbon allowances to trade-exposed sectors. The European Union's Green Deal raises the prospect of free allocation being replaced by a carbon border adjustment mechanism (CBAM) on imported products. This paper provides a simple framework to analyze the competitiveness support provided by these policy instruments. It shows how the rate of carbon leakage can be a "sufficient statistic" to determine the output and profit impacts of the switch to a CBAM. High-leakage sectors will prefer the CBAM while low-leakage sectors will prefer free allocation.

Keywords: Cap-and-trade, carbon border adjustment, carbon leakage, industrial competitiveness

JEL codes: H23 (externalities), L11 (market structure), Q54 (environmental policy)

*My thanks are due to Geoffroy Dolphin for helpful comments. All views expressed and any errors are mine.

1 Introduction

The competitiveness of emissions-intensive trade-exposed (EITE) sectors and mitigating the risk of carbon leakage has been a major concern for sub-global carbon pricing systems like the European Union's emissions trading system (ETS). To date, policy support to EITE sectors has largely been provided by way of free allocation of emissions allowances.

The EU's Green Deal now raises the prospect of free allocation being replaced by a carbon border adjustment mechanism (CBAM) that imposes its carbon price also on imported products (European Commission 2020). Other jurisdictions designing carbon-pricing initiatives will similarly have to confront whether to use free allocation or a CBAM—or both (or neither); border adjustments are already being discussed in countries including the UK (Environmental Audit Committee 2022), the US (Euractiv 2021), and Canada (Government of Canada 2021).

Does a CBAM make free allocation redundant? More broadly, to what extent does a CBAM provide similar—or better—competitiveness support than free allocation? This paper presents a simple economic framework to address these questions at the level of an individual EITE sector like aluminium, cement, petrochemicals or steel. Free allocation tends to reduce the marginal production cost of regulated "inside" (e.g., EU) firms while a CBAM instead raises the marginal cost of "outside" (e.g., non-EU) firms.

The paper shows how the rate of carbon leakage can be a "sufficient statistic" to determine the impact of a switch to a CBAM on the competitiveness of inside firms—as proxied by their production volumes or profit margins. That is, the rate of carbon leakage captures *all* salient features of competition—including market structure and the intensity of rivalry, the degree of product differentiation between inside and outside firms, and the relative carbon intensity of their production. From a political-economy perspective, the analysis suggests that EITE sectors with a modest risk of carbon leakage will lobby to keep a policy regime with free allocation while those with substantial carbon leakage will favour switching to the CBAM.

In short, a CBAM makes free allocation redundant for some sectors—but perhaps not for others. (Cosbey, Droege, Fischer & Munnings (2019) provide a valuable survey of the earlier literature on border carbon adjustments.)

2 Model

Consider competition in an individual EITE sector. There are n_i inside firms and n_j outside firms competing in the product market. The carbon price faced by inside firms in the ETS is τ_i and, for simplicity, assume it is zero for the outside firms.

Production costs per unit of output (excluding carbon costs) are, respectively, c_i for inside firms and c_j for outside firms. Carbon intensities of production are z_i, z_j where $z_k = e_k/x_k$ represents emissions e_k per unit of output x_k (for $k = i, j$). For simplicity, unit costs and carbon intensities are assumed to be constants.

2.1 Policy instruments

Country i has access to two policy instruments: free allocation and carbon border adjustment.

First, an inside firm is subject to the carbon price τ_i on its emissions e_i but also receives a free allocation of A_i which can act as an output subsidy. Its marginal cost of production therefore is:

$$MC_i(f) = c_i + \tau_i \left(\frac{\partial e_i}{\partial x_i} - \frac{\partial A_i}{\partial x_i} \right) = c_i + \tau_i(1-f)z_i \tag{1}$$

where $f = \frac{\partial A_i}{\partial x_i} \Big/ \frac{\partial e_i}{\partial x_i}$ measures the extent of output subsidy at the margin and $\frac{\partial e_i}{\partial x_i} = z_i$ is its carbon intensity.

Free allocation to EITE sectors has to date been generous across most cap-and-trade systems, covering a large fraction of sectoral emissions, that is, $\widetilde{f} \equiv A_i/e_i$ has often been close to 100%. The mode of allocation translates this into the extent of output subsidy f. Grandfathering with $A_i = \phi^G \overline{e}_i$, where $\overline{e}_i$ are historical emissions and $\phi^G \in [0,1]$ is a policy parameter leads to $\widetilde{f} \equiv \phi^G(\overline{e}_i/e_i)$, where typically $\widetilde{f} \leq 1$, but there is no incentive effect $f = 0$. Allocation linked to output via emissions according to $A_i = \phi^B e_i$, where $\phi^B \in [0,1]$, yields $f = \widetilde{f} = \phi^B$ as the marginal and average effects coincide. The EU ETS's hybrid allocation design has combined elements of output-based allocation, grandfathering and a performance standard so that f takes on an intermediate value (Meunier, Ponssard & Quirion 2014).

Second, a border adjustment on imports effectively imposes i's carbon price also on an outside firm. Its marginal cost of production becomes:

$$MC_j(b) = c_j + b\tau_i z_j \tag{2}$$

where b is used as a modelling device to index the strength of the CBAM. The case with $b = 1$ imposes a full border adjustment that makes it as if outside firms were part of the inside firms' ETS. The case with $b = 0$ represents the situation before any CBAM is introduced.

The setup reflects how free allocation reduces inside firms' production costs while a CBAM raises those of outside firms. The central questions in the paper revolve around the degree of "policy substitution" between free allocation (modelled via f) and a CBAM (modelled via b).

2.2 Industrial competition

Consider a reduced-form model of imperfect competition that nests linear versions of familiar models such as Cournot-Nash competition and Bertrand and Cournot competition with differentiated products as special cases.

On the demand side, the inverse demand curves faced by inside firms and outside firms are:

$$p_i(X_i, X_j) = \alpha - \beta X_i - \gamma X_j \text{ and } p_j(X_i, X_j) = \alpha - \beta X_j - \gamma X_i \tag{3}$$

where $\alpha, \beta, \gamma > 0$ are parameters and $X_k = n_k x_k$ are total outputs ($k = i, j$). For an interior solution, assume $\alpha > \max\{MC_i, MC_j\}$. Define $\delta = \gamma/\beta \in [0,1]$ as an inverse index of product differentiation. The corresponding direct demand curves are:

$$D_i(p_i, p_j) = a - bp_i + gp_j \text{ and } D_j(p_i, p_j) = a - bp_j + gp_i \quad (4)$$

where $a, b, g > 0$ are dual parameters of the demand system (Häckner 2000).

On the supply side, suppose that firms behave in the product market according to:

$$x_i = \psi[p_i - MC_i(f)] \text{ and } x_j = \psi[p_j - MC_j(b)] \quad (5)$$

where $\psi > 0$ is a parameter. Given the linear demand structure, these linear supply schedules can be derived from first-order conditions for profit-maximization from various familiar models of competition. For example, $\psi = 1/\beta$ holds under Cournot-Nash competition both with homogeneous ($\delta = 1$) or differentiated ($\delta < 1$) products. Under differentiated-products Bertrand competition, $\psi = b$ is also a constant.

The idea of what follows is to obtain results that hold across different modes of competition, that is, different values of ψ. Given firms' marginal costs $MC_i(f), MC_j(b)$ and these demand- and supply-side conditions, write the resulting market price for inside firms as $p_i = p_i(f, b)$.

3 Carbon cost pass-through

Carbon cost pass-through measures the extent to which carbon costs are reflected in product prices. Define "own-cost" and "cross-cost" pass-through, respectively, for inside firms as:

$$\rho_{ii} \equiv \frac{\partial p_i}{\partial MC_i} \text{ and } \rho_{ij} \equiv \frac{\partial p_i}{\partial MC_j} \quad (6)$$

These definitions are distinct from the pass-through of a *market-wide* uniform cost shift that is often used in the literature. They are more useful in the present context with multiple policy instruments.

Lemma 1. *Own-cost and cross-cost pass-through rates are:*

$$\rho_{ii} = \frac{\beta\psi n_i + (\beta^2 - \gamma^2)\psi^2 n_i n_j}{[(1 + \beta\psi n_i + \beta\psi n_j) + (\beta^2 - \gamma^2)\psi^2 n_i n_j]} \in (0, 1), \textit{ and}$$

$$\rho_{ij} = \frac{\gamma\psi n_j}{[(1 + \beta\psi n_i + \beta\psi n_j) + (\beta^2 - \gamma^2)\psi^2 n_i n_j]} \in (0, 1).$$

Lemma 1 generalizes results from prior literature to encompass a range of different modes of competition (via ψ) as well as asymmetric cost shifts.

In the absence of international competition ($n_j = 0$) or as products become fully differentiated ($\gamma \to 0$), cross-cost pass-through is zero while own-cost pass-through $\rho_{ii} = \frac{\beta\psi n_i}{(1+\beta\psi n_i)}$. In Cournot-Nash equilibrium ($\psi = 1/\beta$), for example, the latter then becomes the familiar $\rho_{ii} = \frac{n_i}{(1+n_i)}$ (Hepburn, Quah & Ritz 2013).

Given the linear demand- and supply-side conditions, the market price for inside firms can be written in a linear form:

$$p_i(f,b) = (1 - \rho_{ii} - \rho_{ij})\alpha + \rho_{ii} MC_i(f) + \rho_{ij} MC_j(b) \tag{7}$$

in terms of the two pass-through coefficients (which are both constant with respect to the carbon price) and where $\rho_{ii} + \rho_{ij} < 1$.

4 Carbon leakage

The rate of carbon leakage measures the extent to which emissions reductions by inside firms are offset by higher emissions from outside firms:

$$L_i^C \equiv -\frac{dE_j}{dE_i} = \frac{z_j}{z_i}\left[-\frac{dX_j}{dX_i}\right] \tag{8}$$

where $E_k = n_k e_k$ are total emissions ($k = i, j$) and the second equality follows because emissions intensities are constant. That is, carbon leakage equals output leakage scaled by the relative emissions intensity of outside firms compared to inside firms.

Lemma 2. *The rate of carbon leakage satisfies:*

$$L_i^C = \frac{z_j}{z_i}\frac{\rho_{ij}}{(1 - \rho_{ii})} > 0.$$

Lemma 2 directly links carbon leakage to pass-through. It makes precise how carbon leakage is positive because cross-cost pass-through is positive. In the absence of international competition ($n_j = 0$) or as products become fully differentiated ($\gamma \to 0$), both metrics go to zero as competitive conditions between inside and outside firms become independent.

Carbon leakage can exceed 100 percent (only) if outside firms are significantly dirtier than inside firms (recalling that $\rho_{ii} + \rho_{ij} < 1$).

5 Border adjustment vs free allocation

The EU's Green Deal raises the prospect of cap-and-trade designs moving from free allocation to a carbon border adjustment.

5.1 Competitiveness

Suppose that the (proximate) policy objective is to maintain "competitiveness" of inside firms. To operationalize this concept, define stable competitiveness as a constant level of inside firms' production, that is, policies change in such a way that $dX_i = 0$. By the linear supply-side schedules, this corresponds to stable profitability: $dX_i = 0 \Leftrightarrow d[p_i - MC_i] = 0$. It also leads to constant emissions by inside firms, $dE_i = 0$, so coincides with a local climate-policy objective.

Proposition 1. *The rate of policy substitution from free allocation to a CBAM that yields stable competitiveness for inside firms is given by:*

$$\left.\frac{df}{db}\right|_{dX_i=0} = -L_i^C.$$

Proposition 1 establishes a remarkably sharp result, with the rate of carbon leakage as a "sufficient statistic" (Chetty 2009) for iso-competitiveness. It captures *all* salient features of competition—including market structure (n_j) competitive behaviour (ψ), the degree of product differentiation between inside and outside firms (δ), and their relative carbon intensities (z_j/z_i). This insight holds across a range of different modes of imperfect competition.

The reason for the result is as follows. On one hand, a small decrease $df < 0$ in inside firms' free allocation affects their profitability by $\left[\frac{\partial p_i}{\partial f} - \frac{\partial MC_i}{\partial f}\right] df$. This also writes as $[\tau_i z_i(1-\rho_{ii})]\,df$ using the definition of the own-cost pass-through rate $\rho_{ii} \equiv \frac{\partial p_i}{\partial MC_i}$ and that free allocations lowers i's costs by $\frac{\partial MC_i}{\partial f} = -\tau_i z_i$.

On the other hand, a small tightening $db > 0$ of a CBAM for outside firms helps inside firms' profitability according to $\frac{\partial p_i}{\partial b} db$ (as MC_i remains unchanged). This also writes as $[\tau_i z_j \rho_{ij}]\,db$ using the definition of the own-cost pass-through rate $\rho_{ij} \equiv \frac{\partial p_i}{\partial MC_j}$ and that the CBAM raises j's costs by $\frac{\partial MC_j}{\partial b} = \tau_i z_j$.

Proposition 1 gives the condition for these two effects to exactly offset one another, leading to stable competitiveness. And the resulting expression is the rate of carbon leakage of Lemma 2. Both effects scale linearly with the level of the carbon price τ_i which therefore does not matter for the rate of policy substitution.

In a nutshell, a tighter CBAM enables a stronger reduction in free allocation—while maintaining stable competitiveness—when carbon leakage is stronger. This is intuitive: a sector with weak carbon leakage benefits little from a CBAM as it stands only in weak competition against outside firms; conversely, for a sector with high carbon leakage, the CBAM has much more "bite".

So does a CBAM make free allocation redundant? Suppose that the initial policy support is given by $(f, b) = (f_0, 0)$, where $f_0 \in (0, 1]$ is the initial level of free allocation and there is no CBAM. Now there is a policy proposal to discontinue free allocation and instead introduce a CBAM, that is, move to $(f, b) = (0, 1)$.

Corollary 1. *If policy support moves from free allocation* $(f, b) = (f_0, 0)$ *to a CBAM* $(f, b) = (0, 1)$*, this increases the competitiveness of inside firms* ($dX_i \geq 0$) *if and only if*

$$L_i^C \geq f_0.$$

Corollary 1 resolves the trade-off between the two policy instruments using the result from Proposition 1: the CBAM makes free allocation redundant for some sectors—but not for others.

The shift to a CBAM is more likely to enhance competitiveness if the sector has higher carbon leakage or a lower free allocation in the first place. Any sector with carbon leakage in excess of 100% experiences enhanced competitiveness from the CBAM, as does any sector that had a grandfathered allowance allocation.

Empirical estimates of carbon leakage for individual EITE sectors vary widely, ranging from "low" estimates up to 20% to "medium" estimates around 50% and some "high" estimates that exceed 100% (see Karp 2010 for an informative discussion, including on how partial-equilibrium approaches often yield higher leakage rates than general-equilibrium models).

The EU ETS has involved generous allocation levels $\widetilde{f}_0$ often close to 100%, with its hybrid allocation mode likely leading to "medium" values of f_0 (Meunier, Ponssard & Quirion 2014).

These arguments suggests that the condition of Corollary 1 may resolve in opposite directions for different EITE sectors in different cap-and-trade systems. If the condition is not met, then a continued-but-smaller free allocation would be needed to preserve competitiveness under the CBAM at the level of the status quo ante.

5.2 Profits

The analysis so far has focused on stable output (and profit margins) of inside firms as a competitiveness metric. A related metric is the impact of the switch to a CBAM on inside firms' profits. The profit impact will hinge on the details of the mode of allowance allocation:

Proposition 2. *Let policy support switch from free allocation ($b = 0$) to a CBAM ($b = 1$).*
(a) With output-based allocation $A_i = \phi^B e_i$, where $\phi^B \in [0, 1]$ is a policy parameter, then $f_0 = \widetilde{f}_0 = \phi^B$ and the profits of inside firms increase ($d\Pi_i \geq 0$) if and only if

$$L_i^C \geq \phi^B,$$

which coincides with the condition for increased competitiveness.
(b) With grandfathered allocation $A_i = \phi^G \overline{e}_i$, where $\phi^G \in [0, 1]$ is a policy parameter, then $f_0 = 0, \widetilde{f}_0 = \phi^G$ and, to first order, the profits of inside firms increase ($d\Pi_i \geq 0$) if and only if

$$2(1 - \rho_{ii})L_i^C \geq \Gamma_i \phi^G,$$

where $\Gamma_i = [\overline{e}_i/e_i(0, 0)]$ is the ratio of historical emissions to actual emissions under grandfathering, while their competitiveness always increases.

The condition for output-based free allocation to raise profits in Proposition 2(a) is identical to that for competitiveness to increase with the CBAM in Proposition 1; a sector with relatively high leakage will prefer the CBAM to this mode of free allocation. Here, a firm's profits increase if only if its output increases as the marginal and average effects of free allocation coincide ($d\Pi_i \geq 0 \Leftrightarrow dX_i \geq 0$). Figure 1 illustrates these results for two cases of (f_0, L_i).

The condition for grandfathering has a similar flavour but different detail. Here, free allocation has no incentive effect so dropping it in favour of the CBAM always raises competitiveness. However, the impact on profits also takes into account the lost lump-sum value of the free allocation. Proposition 2(b) distills this comparison: if carbon leakage is sufficiently high and

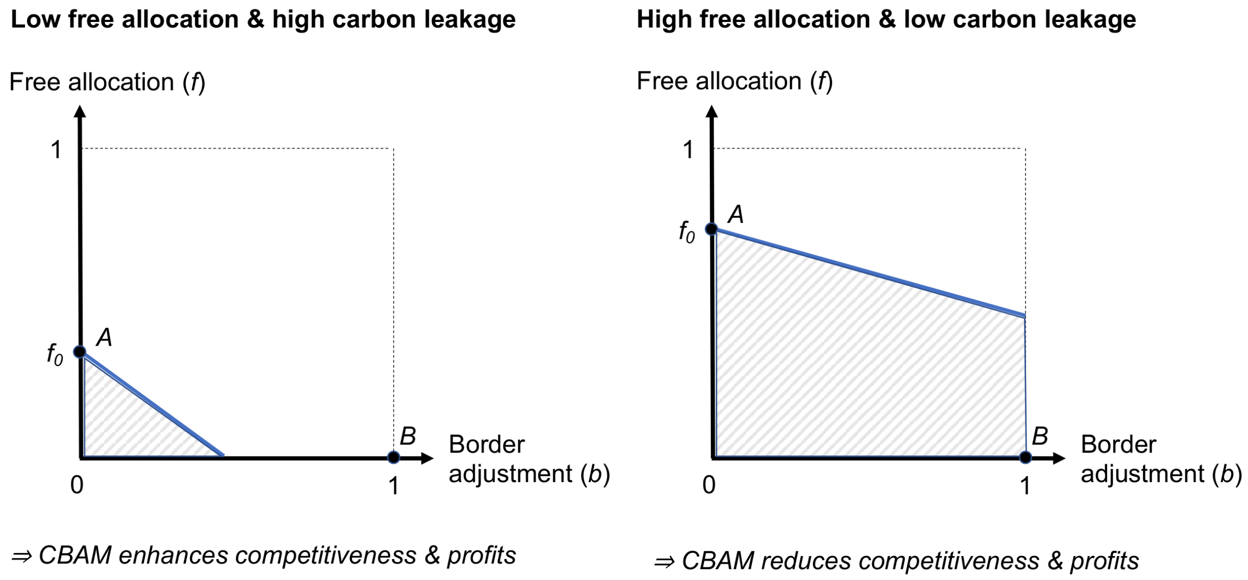

Figure 1: Illustration of impacts of a switch from output-based free allocation to CBAM on competitiveness (Corollary 1) and profits (Proposition 2(a))

Notes: Competitiveness and profits decline in the shaded grey area. In the left-hand panel, they are higher at point B (CBAM) than at point A (initial free allocation); in the right-hand panel, this ordering is reversed. The slope of the blue line is equal to minus the rate of carbon leakage.

own-cost pass-through sufficiently low, the profit gain from the CBAM being levied on relatively high-emissions outside firms outweights the profit loss from discontinued free allocation. (The simple formula is a first-order result that holds exactly for a small carbon price and is otherwise an approximation.)

From a political-economy perspective, a basic prediction is that—if forced to choose between the two instruments—high-leakage sectors will lobby for the CBAM while low-leakage sectors will lobby to keep free allocation.

6 Conclusion and policy implications

The question of how to best support EITE sectors such as cement and steel on the road to decarbonization is of increasing policy importance.

This paper has introduced a simple framework to understand the switch from free allocation to a border adjustment, and has shown how the rate of carbon leakage can be a sufficient statistic to determine the competitiveness and profit impacts of the switch to a CBAM. The results suggest that sectors with "high" leakage will prefer the CBAM while those with "low" leakage will prefer free allocation. These findings could be calibrated empirically in future work.

An advantage of the framework presented is its simplicity and application across a range of models of competition amongst which it can be difficult to choose (e.g., Cournot vs Bertrand). Limitations include the absence of abatement (other than output reductions) and the focus on competitiveness support as a proximate policy objective (rather than social welfare). Nonetheless it seems likely that the rate of carbon leakage will play a central role also in richer models.

References

Chetty, Raj (2009). Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods. *Annual Review of Economics* 1, 451–87.

Cosbey, Aaron, Susanne Droege, Carolyn Fischer and Clayton Munnings (2019). Developing guidance for implementing border carbon adjustments: Lessons, cautions, and research needs from the literature. *Review of Environmental Economics and Policy* 13(1), 3–22.

Environmental Audit Committee (2022). Greening imports: A UK carbon border approach, Fifth Report of Session 2021-22. House of Commons, 23 March 2022.

Euractiv (2021). US lawmakers push carbon border tariff similar to EU's CBAM. Kira Taylor, Euractiv.com, 21 July 2021.

European Commission (2020). Inception impact assessment: Carbon border adjustment mechanism. Ref. Ares(2020)1350037, 4 March 2020.

Government of Canada (2021). Budget 2021: A recovery plan for jobs, growth, and resilience. April 2021.

Häckner, Jonas (2000). A note on price and quantity competition in differentiated oligopolies. *Journal of Economic Theory* 93, 233–239.

Hepburn, Cameron J., John K.-H. Quah and Robert A. Ritz (2013). Emissions trading with profit-neutral permit allocations. *Journal of Public Economics* 98, 85–99.

Karp, Larry (2010). Reflections on carbon leakage. Working paper at University of California at Berkeley, 13 October 2010.

Meunier, Guy, Jean-Pierre Ponssard and Philippe Quirion (2014). Carbon leakage and capacity-based allocations: Is the EU right? *Journal of Environmental Economics and Management* 68, 262–279.

Appendix: Proofs of results

Proof of Lemma 1. Aggregating the two linear supply-side conditions $x_i = \psi[p_i - MC_i(f)]$ and $x_j = \psi[p_j - MC_j(b)]$ over all n_i inside and n_j outside firms respectively gives:

$$X_i = \psi n_i[p_i - MC_i(f)] \text{ and } X_j = \psi n_j[p_j - MC_j(b)] \tag{9}$$

Using these expressions in the demand curve for inside firms $p_i(X_i, X_j) = \alpha - \beta X_i - \gamma X_j$ then yields:

$$p_i = \alpha - \beta\psi n_i[p_i - MC_i(f)] - \gamma\psi n_j[p_j - MC_j(b)] \tag{10}$$

which rearranges to:

$$p_i = \frac{\alpha + \beta\psi n_i MC_i(f) + \gamma\psi n_j MC_j(b) - \gamma\psi n_j p_j}{(1 + \beta\psi n_i)} \tag{11}$$

The same procedure for the outside firms' demand curve yields a mirror-image expression:

$$p_j = \frac{\alpha + \beta\psi n_j MC_j(b) + \gamma\psi n_i MC_i(f) - \gamma\psi n_i p_i}{(1 + \beta\psi n_j)} \tag{12}$$

Solving the resulting system of two equations and two unknowns gives inside firms' market price as:

$$\begin{aligned} p_i \quad &= \quad \frac{\alpha + \beta\psi n_i MC_i(f) + \gamma\psi n_j MC_j(b) - \frac{\gamma\psi n_j}{(1+\beta\psi n_j)}\left[\alpha + \beta\psi n_j MC_j(b) + \gamma\psi n_i MC_i(f)\right]}{(1 + \beta\psi n_i)} \\ &\quad + \frac{\gamma\psi n_j}{(1 + \beta\psi n_i)}\frac{\gamma\psi n_i}{(1 + \beta\psi n_j)} p_i \end{aligned} \tag{13}$$

or

$$\begin{aligned} p_i \quad &= \quad \frac{\alpha\left(1 + (\beta - \gamma)\psi n_j\right) + MC_i(f)\left(\beta\psi n_i(1 + \beta\psi n_j) - \gamma\psi n_j\gamma\psi n_i\right)}{[(1 + \beta\psi n_i)(1 + \beta\psi n_j) - \gamma\psi n_i\gamma\psi n_j]} \\ &\quad + \frac{MC_j(b)\left(\gamma\psi n_j(1 + \beta\psi n_j) - \gamma\psi n_j\beta\psi n_j\right)}{[(1 + \beta\psi n_i)(1 + \beta\psi n_j) - \gamma\psi n_i\gamma\psi n_j]} \end{aligned} \tag{14}$$

The two pass-through rates now follow immediately by differentiation as:

$$\rho_{ii} \equiv \frac{\partial p_i}{\partial MC_i} = \frac{(\beta\psi n_i(1 + \beta\psi n_j) - \gamma\psi n_j\gamma\psi n_i)}{[(1 + \beta\psi n_i)(1 + \beta\psi n_j) - \gamma\psi n_i\gamma\psi n_j]} = \frac{\beta\psi n_i + (\beta^2 - \gamma^2)\psi^2 n_i n_j}{[(1 + \beta\psi n_i + \beta\psi n_j) + (\beta^2 - \gamma^2)\psi^2 n_i n_j]} \in (0, 1)$$

$$\rho_{ij} \equiv \frac{\partial p_i}{\partial MC_j} = \frac{(\gamma\psi n_j(1 + \beta\psi n_j) - \gamma\psi n_j\beta\psi n_j)}{[(1 + \beta\psi n_i)(1 + \beta\psi n_j) - \gamma\psi n_i\gamma\psi n_j]} = \frac{\gamma\psi n_j}{[(1 + \beta\psi n_i + \beta\psi n_j) + (\beta^2 - \gamma^2)\psi^2 n_i n_j]} \in (0, 1)$$

which completes the proof.

Proof of Lemma 2. Differentiating the aggregated supply-side condition for j's firms $X_j =$

$\psi n_j[p_j - MC_j(b)]$ yields:

$$\frac{dX_j}{dX_i} = \psi n_j \left(\frac{dp_j}{dX_j}\frac{dX_j}{dX_i} + \frac{dp_j}{dX_i} \right) = \psi n_j \left(-\beta \frac{dX_j}{dX_i} - \gamma \right) \tag{15}$$

where the second equality uses j's demand curve $p_j(X_i, X_j) = \alpha - \beta X_j - \gamma X_i$. Hence the rate of carbon leakage satisfies:

$$L_i^C \equiv -\frac{dE_j}{dE_i} = \frac{z_j}{z_i}\left[-\frac{dX_j}{dX_i} \right] = \frac{z_j}{z_i}\frac{\gamma\psi n_j}{(1+\beta\psi n_j)} \tag{16}$$

But note from Lemma 1 that own-cost pass-through satisfies:

$$1 - \rho_{ii} = \frac{(1+\beta\psi n_j)}{[(1+\beta\psi n_i)(1+\beta\psi n_j) - \gamma\psi n_i \gamma\psi n_j]} \tag{17}$$

so that also

$$L_i^C = \frac{z_j}{z_i}\frac{\rho_{ij}}{(1-\rho_{ii})} \tag{18}$$

as claimed.

Proof of Proposition 1. Stable competitiveness corresponds to $dX_i = 0 \Leftrightarrow d[p_i - MC_i] = 0$ while varying the two policy instruments (f, b). Hence differentiating the latter condition gives:

$$[dp_i(f,b) - dMC_i(f)] = 0 = \frac{\partial p_i}{\partial f}df + \frac{\partial p_i}{\partial b}db + \frac{\partial MC_i}{\partial f}df \tag{19}$$

and so rearranging for the rate of policy substitution yields:

$$\frac{df}{db} = -\frac{\frac{\partial p_i}{\partial b}}{\left(\frac{\partial p_i}{\partial f} - \frac{\partial MC_i}{\partial f}\right)} = -\frac{\rho_{ij}}{(1-\rho_{ii})}\frac{z_j}{z_i} = L_i^C. \tag{20}$$

where the second equality uses the expression for the market price $p_i(f,b)$ which shows that $\frac{\partial p_i}{\partial f} = \rho_{ii}\frac{\partial MC_i}{\partial f}$ while $\frac{\partial p_i}{\partial b} = \rho_{ij}\frac{\partial MC_j}{\partial b}$ and that the costs terms in turn satisfy $\frac{\partial MC_i}{\partial f} = -\tau_i z_i < 0$ and $\frac{\partial MC_j}{\partial b} = \tau_i z_j > 0$ and the third equality uses Lemma 2.

Proof of Corollary 1. By Proposition 1, the introduction of the CBAM is "worth" a reduced free allocation of $\int_{s=0}^{1} \frac{df}{db}\Big|_{dX_i=0}(s) = -L_i^C$. So the inside firms' competitiveness increases if and only if $L_i^C \geq f_0$, as claimed.

Proof of Proposition 2. In general, for a free allocation characterized by $(\widetilde{f}, f)$ and writing price as $p_i(f, 0)$ the profits of an individual (symmetric) inside firm are:

$$\begin{aligned} \Pi_i(\widetilde{f}, f) &= [p_i(f,0) - c_i]\, x_i - \tau_i e_i + \tau_i A_i \\ &= [p_i(f,0) - c_i]\, x_i - (1-\widetilde{f})\tau_i e_i \qquad (21) \\ &= [p_i(f,0) - MC_i(f)]\, x_i + (\widetilde{f} - f)\tau_i e_i, \qquad (22) \end{aligned}$$

where the second line uses $\widetilde{f} \equiv A_i/e_i$ and the third line uses $MC_i(f) = c_i + \tau_i(1-f)z_i$ and some rearranging. Using the linear supply schedule $x_i = \psi[p_i - MC_i(f)]$, profits are also:

$$\Pi_i(\widetilde{f}, f) = \psi[p_i(f,0) - MC_i(f)]^2 + (\widetilde{f} - f)\tau_i e_i. \tag{23}$$

For part (a), with an output-linked allocation $A_i = \phi^B e_i$, where $\phi^B \in [0,1]$, then $f = \widetilde{f} = \phi^B$ and so $\Pi_i = \psi[p_i(f,0) - MC_i(f)]^2$. Hence profits increase with the switch to a CBAM if and only if the profit margin increases—which is exactly Proposition 1. So the condition for the switch from free allocation to a CBAM follows directly from Corollary 1.

For part (b), with a grandfathered allocation $A_i = \phi^G \overline{e}_i$, where $\phi^G \in [0,1]$, then $f = 0, \widetilde{f} = \phi^G(\overline{e}_i/e_i)$ and so profits with free allocation are:

$$\Pi_i(\phi^G) = \left[\psi[p_i(0,0) - MC_i(0)]^2 + \phi^G \tau_i \overline{e}_i\right]. \tag{24}$$

while profits under the CBAM ($b = 1$) are:

$$\Pi_i(b)|_{b=1} = \psi[p_i(0,1) - MC_i(0)]^2 \tag{25}$$

So profits under the CBAM are higher if and only if:

$$\Pi_i(b)|_{b=1} \geq \Pi_i(\phi^G) \Leftrightarrow \psi[p_i(0,1) - MC_i(0)]^2 \geq \left[\psi[p_i(0,0) - MC_i(0)]^2 + \phi^G \tau_i \overline{e}_i\right] \tag{26}$$

To first order (omitting second-order terms in τ_i), and letting $p_i(0,1) = p_i(0,0) + \Delta p_i(0,1)$, this comparison boils down to:

$$2\psi[p_i(0,0) - MC_i(0)]\Delta p_i(0,1) \geq \phi^G \tau_i \overline{e}_i \tag{27}$$

As $\frac{\partial p_i}{\partial b} = \rho_{ij}\frac{\partial MC_j}{\partial b} = \rho_{ij}\tau_i z_j > 0$ from the proof of Prooposition 1, it follows that $\Delta p_i(0,1) = \int_{s=0}^{1} \frac{\partial p_i}{\partial b}(s)ds = \rho_{ij}\tau_i z_j$ and recalling that $[p_i(0,0) - MC_i(0)] = \frac{x_i(0,0)}{\psi}$ the comparison also writes as:

$$2x_i(0,0)\rho_{ij}z_j \geq \phi^G \overline{e}_i \tag{28}$$

noting that both the carbon price τ_i and the supply-schedule parameter ψ cancel out. Writing this in terms of carbon leakage $L_i^C = \frac{z_j}{z_i}\frac{\rho_{ij}}{(1-\rho_{ii})}$ from Lemma 2 gives:

$$2(1-\rho_{ii})L_i^C \geq \left(\frac{\overline{e}_i}{z_i x_i(0,0)}\right)\phi^G = \Gamma_i \phi^G \tag{29}$$

where $\Gamma_i = [\overline{e}_i/e_i(0,0)]$ is the ratio of historical emissions to actual emissions under grandfathering, thus completing the proof.